GOVERNING RELATIONSHIPS OF DYNAMICAL PLASTICITY

N. S. Kozin, A. A. Tuzovskii, and N. N. Kholin

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Interpolation formulas are constructed in this paper for the elastic energy of a number of metals [Fe(α-phase), Al, Cu, Ni, Pb, Ti, Be, Au, Cd, Pt, Ag, Zn, Mg, Sn, Sb, V, Nb, Ta, Mo, W, Co, Th, U, Zr] which take account of the molding and defect formation in the crystal lattice. To do this, a dependence on the second and third order invariants of the elastic strain tensor as well as on a scalar parameter characterizing the defect density of the crystal lattice is introduced in the internal energy equation of the medium. The formulas are valid in the compression range $0.6 \leq \rho^*/\rho \leq 1$ and the temperature range $0 \leq T(K) \leq \Theta_e$, where Θ_e is the temperature of electron degeneration. The interpolation coefficients for the elastic energy are compared to the moduli of second approximation elasticity theory. On the basis of the equations of state constructed, the magnitude of the works of the forces in the plastic strains which go into the formation of defects is estimated.

§1. To close the complete system of mechanical equations when the effects of a change in temperature during elastoplastic strain of a solid are important, it is necessary to examine the thermodynamic relationships in the theory of dynamical plasticity. It should hence be kept in mind that the plastic deformation results in a change in the magnitude of the material internal energy because of the change in its structure in a number of cases important for practical applications. Thus, a discontinuity in the internal intramolecular bonds occurs in macropolymers during deformation, cumulative dislocations occur in metals under plastic deformation, etc. Hence, a dependence on a certain storage parameter characterizing the density of the defects being formed must be taken into account in the dependence of the internal energy on the elastic strain and the entropy parameter which takes account of the internal energy redistribution because of heat liberation. In the general case, this parameter is a tensor. Therefore, by determining the model of the viscoelastoplastic body, the density of the internal energy can be written as

$$E = E(\varepsilon_{ij}^e, S, n_{ij}), \tag{1.1}$$

where ε_{ij}^e is the elastic strain tensor, S is the entropy per unit mass of the medium, n_{ij} is the tensor of the defects (or the tensor of the dislocation density [1]). The second law of thermodynamics has the form

$$TdS = dq' + dq^e,$$

where T is the absolute temperature, dq' is the uncompensated heat [2], and $dq^{(e)}$ is the external heat inflow. We have the following assumption:

$$E(\varepsilon_{ij}^e, S, n_{ij}) = E_e(\varepsilon_{ij}^e, S) + E_d(n_{ij}) + E_*, \tag{1.2}$$

for the dependence (1.1), where E_* is a normalizing constant.

This assumption is given a foundation by the requirement that the stress tensor σ_{ij}, which is defined as follows [2]:

$$\sigma_{ij} = \rho \frac{\partial E}{\partial \varepsilon_{ij}^e}, \tag{1.3}$$

would be independent of the parameter n_{ij} but would be defined only by the elastic internal bonds* [ρ is the density of the medium in (1.3)].

*The Murnaghan formulas [3]

$$\sigma_{ij} = \rho(\delta_{i\alpha} - 2\varepsilon_{i\alpha})\partial E/\partial \varepsilon_{\alpha j}$$

should be used in investigating finite strains.

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§2. Following [4], we give the elastic part of the internal energy of an isotropic medium in the form

$$E_e = E_p + E_t, \tag{2.1}$$

where E_p and E_t are, respectively, the cold and warm components of the internal energy. The dependence of the internal energy on the elastic strains will be taken into account as follows. Let $\varepsilon_i^e (i = x, y, z)$ denote the logarithm of the elastic elongations λ_i $(i = x, y, z)$ along the principal elastic strain axes x, y, z:

$$\varepsilon_x^e = \ln \lambda_x, \quad \varepsilon_y^e = \ln \lambda_y, \quad \varepsilon_z^e = \ln \lambda_z. \tag{2.2}$$

Symmetric functions of ε_i^e: ε, D, and Δ of the form

$$\varepsilon = -(\varepsilon_x^e + \varepsilon_y^e + \varepsilon_z^e) = \ln \rho/\rho^0, \quad D = \frac{1}{2}(d_x^2 + d_y^2 + d_z^2),$$
$$\Delta = \frac{1}{3}(d_x^3 + d_y^3 + d_z^3), \quad d_x = \varepsilon_x^e + \frac{\varepsilon}{3}, \quad d_y = \varepsilon_y^e + \frac{\varepsilon}{3}, \quad d_z = \varepsilon_z^e + \frac{\varepsilon}{3}$$

are invariants of the elastic strain tensor ε_{ij}^e. Let us use the representation of the cold part of the internal energy of an isotropic medium in the form of polynomials in powers of the invariants ε, D, and Δ:

$$E_p = h(\varepsilon) + 2d^2(\varepsilon)D + \overline{M}\Delta,$$
$$h(\varepsilon) = \frac{1}{2}a_0^2\varepsilon^2(1 + l_1\varepsilon + l_2\varepsilon^2), \tag{2.3}$$
$$d^2(\varepsilon) = d_0^2(1 + l_3\varepsilon + l_4\varepsilon^2 + l_5\varepsilon^3),$$

where d_0, a_0 are the velocities of the transverse and volume waves at $\varepsilon = 0$, $T = 0$; the constants $l_1 - l_5$, $\overline{M}$ have the meaning of interpolation coefficients. The values of the cold pressure p_p and the volume compression modulus K are calculated as follows

$$p_p = \rho \frac{\partial E}{\partial \varepsilon}\Big|_{D=\Delta=0} = \rho a_0^2\left(1 + \frac{3}{2}l_1\varepsilon + 2l_2\varepsilon^2\right)\varepsilon,$$
$$K(\varepsilon) = a^2 = \frac{1}{\rho}\frac{\partial p_0}{\partial \varepsilon} = a_0^2\left[1 + (1 + 3l_1)\varepsilon + \left(\frac{3}{2}l_1 + 6l_2\right)\varepsilon^2 + 2l_2\varepsilon^3\right].$$

The longitudinal speed of sound c equals $\sqrt{a^2 + (4/3)d^2}$, while the square of the transverse velocity is independent of the temperature under the conditions of the given interpolation and is given by the expression

$$d^2 = \frac{1}{2}\frac{\partial E}{\partial D} = d_0^2(1 + l_3\varepsilon + l_4\varepsilon^2 + l_5\varepsilon^3).$$

The third-order elastic modulus $\overline{M}$ is considered constant.

The member E_t representing the thermal part of the energy is given parametrically in terms of the entropy parameter $s > 0$

$$E_t = \Theta(\varepsilon)g(s) + g(s_0)\Theta_0 l_6\rho^0 e^{-\varepsilon}.$$

The Debye temperature θ, the absolute temperature T, the entropy S, and the function $q = c_c T/\theta$ which equals the product of the specific heat by the reduced temperature, are given functions

$$\Theta(\varepsilon) = \Theta_0(1 + l_6\varepsilon + l_7\varepsilon^2 + l_8\varepsilon^3), \quad T = \Theta(\varepsilon)s.$$

In order for the functions S(s) and g(s) to be constructed as easily as possible, the methods of calculating them presented next, should be used. When $s = T/\theta < 1$

$$g(s) = \frac{3R}{\mu_a}\left[\frac{3}{8}\frac{\pi^4}{5}s^4 - 3s\sum_{k=1}^{N-1}\frac{1}{k}e^{-\frac{k}{s}}\left(1 + 3\frac{s}{k} + 6\left(\frac{s}{k}\right)^2 + 6\left(\frac{s}{k}\right)^3\right)\right] + \frac{9R}{\mu_a}\overline{D}(N, s),$$

$$S(s)=\frac{4R\pi^4}{5\mu_a}s^3-\frac{9R}{\mu_a}\sum_{k=1}^{N-1}\frac{e^{-\frac{k}{s}}}{k}\left(1+4\frac{s}{k}+8\left(\frac{s}{k}\right)^2+8\frac{s^3}{k^3}\right)+\frac{3R}{\mu_a}\hat{D}(N,s),$$

where the remainder terms $\bar{D}$ and $\hat{D}$ admit of the estimate

$$|\bar{D}|<\frac{6e^{-\frac{N}{s}+\frac{1}{s}}}{\left(e^{\frac{1}{s}}-1\right)N}\frac{1-(s/N)^4}{1-(s/N)},\quad |\hat{D}|<\frac{8e^{-\frac{N-1}{s}}}{\left(e^{\frac{1}{s}}-1\right)N}\frac{1-(s/N)^4}{1-(s/N)}.$$

For sufficiently high temperatures ($s = T/\Theta \geq 1$), it is convenient to use the formulas

$$g(s)=\frac{9R}{\mu_a}\sum_{k=0}^{N-1}\frac{B_{2k}}{(2k+3)(2k)!}s^{1-2k}+\bar{H}(N,s),$$

$$S(s)=\frac{3R}{\mu_a}(1+\ln s)+\frac{9R}{\mu_a}\sum_{k=1}^{N-1}\frac{B_{2k}(2k-1)}{(2k)(2k+3)(2k)!}s^{-2k}+\hat{H}(N,s),$$

where

$$|\bar{H}(N,s)|<\frac{9R}{\mu_a}\frac{|B_{2N}|}{(2N-3)(2N)!}s^{1-2N};\quad |\hat{H}(N,s)|<\frac{9R}{\mu_a}\frac{|B_{2N}|(2N-1)}{2N(2N+3)(2N)!}s^{-2N}.$$

Here μ_a is the atomic weight, $R = 8.31\cdot10^7$ g·cm/(sec²·deg·mole), and the coefficients B_{2k} are the Bernoulli numbers [4]: $B_0 = 1$, $B_2 = 1/6$, $B_4 = -1/30$, etc.

The calculations of g(s) and S(s) were performed by means of the formulas mentioned, which were compiled in the form of standard procedures with automatic selection of the number of terms for a given absolute error. Rapid convergence of the series permitted the actual use of a small number of terms, for instance, the absolute value of the remainder terms did not exceed 10^{-6} for N = 3.

For the defect part E_d of the elastic energy, we use the representation

$$E_d = b^2d^2n,$$

which expresses the energy of a dislocation concentration per unit mass [5]. The scalar quantity n is the defect density (the dimension of n is cm^{-2}), b is the Burgers vector, and d is the velocity of shear wave propagation.

The normalizing constant E_* in (1.2) was selected from the considerations that E = 0 in the undeformed state under normal conditions, whereupon $E_* = \Theta_0 g(s_0)(1 + l_6)$.

By using the relationship

$$\sigma_i=-\rho\frac{\partial E}{\partial\varepsilon}+\rho d_i\frac{\partial E}{\partial D}-\rho\left(\frac{2}{3}D-d_i^2\right)\frac{\partial E}{\partial\Delta}$$

the reduced elastic energy equation permits computation of the stress state of the medium (i = x, y, z)

$$\sigma_i=-p+2\rho d^2d_i-\rho\left(\frac{2}{3}D-d_i^2\right)\overline{M},$$

$$p=a_0^2\rho\left(1+\frac{3}{2}l_1\varepsilon+2l_2\varepsilon^2\right)+g(s)\rho\Theta_0(l_6+2l_7\varepsilon+3l_8\varepsilon^2)-\rho_0 g(s_0)\Theta_0 l_6.$$

Values of the coefficients of the internal energy interpolation formulas constructed above are presented in Tables 1-4 for the following metals: Fe (α-phase), Al, Cu, Ni, Pb, Ti, Be, Au, Cd, Pt, Ag, Zn, Mg, Sn, Sb, V, Nb, Ta, Mo, W, Co, Th, U, and Zr.

§3. The Born–Mayer potential

$$E_p=\frac{3A}{B\rho^0}\exp[B(1-\delta^{-1/3})]-\frac{3\varkappa}{\rho^0}\delta^{1/3},\quad \delta=\rho/\rho^0,$$

was taken as the initial cold energy equation in constructing the interpolation dependences

TABLE 1

Material	ρ^*, g/cm^3	$c_0 \cdot 10^{-6}$, cm/sec	$d_0 \cdot 10^{-6}$, cm/sec	$a_0^2 \cdot 10^{-12}$, cm^2/sec	Θ_0, K	μ_a, g/mole
Fe	7,84	0,5694	0,2866	0,2147	420	55,85
Al	2,785	0,6122	0,2941	0,2594	390	26,98
Cu	8,9	0,4651	0,2141	0,1552	315	63,54
Ni	8,86	0,5437	0,2485	0,2133	375	58,71
Pb	11,34	0,2151	0,0812	0,0375	88	207,21
Ti	4,51	0,5858	0,2963	0,2261	380	47,90
Be	1,845	1,289	0,89543	0,5925	1000	9,013
Au	19,24	0,324	0,10704	0,0897	170	197,0
Cd	8,64	0,278	0,12941	0,05496	120	112,41
Pt	21,37	0,396	0,13083	0,13399	230	195,09
Mg	1,725	0,575	0,30731	0,2047	318	24,32
Sn	7,28	0,332	0,16416	0,0743	260	118,70
Ag	10,5	0,37	0,17199	0,09746	215	107,88
Zn	7,135	0,417	0,25666	0,08606	235	65,38
Sb	6,67	0,342	0,21059	0,05783	200	121,76
V	6,1	0,6	0,27775	0,25714	273	50,95
Nb	8,6	0,49	0,17325	0,20008	280	92,91
Ta	16,46	0,34	0,07967	0,10712	225	180,95
Mo	10,2	0,619	0,30241	0,26144	380	95,95
W	19,17	0,5174	0,28446	0,15981	310	183,86
Co	8,82	0,472	0,06696	0,21681	385	58,94
Th	11,68	0,29	0,17134	0,04496	100	232,05
U	18,9	0,315	0,1619	0,06428	160	235
Zr	6,49	0,38	0,087037	0,1343	250	91,22

(2.1), where the values of the constants are presented for a number of metals in [4]. The expression for the cold energy permits calculation of the volume compression modulus

$$K(\delta) = \frac{\partial}{\partial \delta}\left(\delta^2 \frac{\partial E_p}{\partial \delta}\right) \tag{3.1}$$

and the Debye temperature [4]

$$\Theta(\delta) = \Theta_0 \delta^{\xi+1/3} \sqrt{K(\delta)/K(1)}. \tag{3.2}$$

According to [6], where use of the relationship between the longitudinal and transverse wave velocities resulting from the Debye theory

$$1/c^3 + 2/d^3 = L\delta/\Theta^3(\delta) \tag{3.3}$$

is proposed (L is a certain constant), the transverse wave velocity is then calculated.

On the basis of the relationships (3.1)-(3.3), tables of the functions $K(\varepsilon)$, $d^2(\varepsilon)$, and $\gamma(\varepsilon) = \Theta(\varepsilon)/\Theta_0$ are compiled in the range $0.6 \leq e^{-\varepsilon} \leq 1$, which were approximated by cubic polynomials in ε. By using the quadratures

$$p_p = \int_0^{\varepsilon} \rho K(\varepsilon)\, d\varepsilon, \quad E_p = \int_0^{\varepsilon} \frac{p_p}{\rho}\, d\varepsilon + 2d^2(\varepsilon) D + \overline{M}\Delta$$

the cold energy $E_p(\varepsilon, D, \Delta)$ was restored, and whose global part was then again interpolated by a fourth power polynomial in ε [formula (2.3)]. To find the constant $\overline{M}$, data presented in [7] and obtained by measuring the third-order elastic modulus by ultrasonic methods were used. Values of the Murnaghan elastic moduli $\overline{l}$, $\overline{m}$, and $\overline{n}$ are presented in Table 3 for a number of metals, which had been determined by expanding the internal energy E in the strain tensor invariants I_1, I_2, I_3:

TABLE 2

Material	l_1	l_2	l_3	l_4	l_5	l_6	l_7	l_8
Fe	0,61316	0,1819	2,6809	1,4574	—0,045029	1,6774	0,35916	0,13524
Al	1,089	0,9191	3,5566	3,0028	0,66827	2,1227	0,65135	0,28279
Cu	0,9657	0,6596	3,3876	2,7782	0,64406	2,0321	0,63782	0,26862
Ni	1,4402	2,1385	9,0771	—3,5491	141,57	4,0408	4,7945	10,012
Pb	0,9126	0,5436	5,2163	5,24	22,145	2,7905	2,2017	2,1319
Ti	3,2763	14,4552	1,3142	—3,9454	2,6334	1,1416	—2,0709	1,247
Be	2,3408	6,2564	1,3095	—4,0019	2,7124	1,1349	—1,7525	0,8685
Au	0,7331	0,2935	5,9804	4,8636	43,158	3,0739	3,0576	3,9354
Cd	—0,2004	—0,1098	3,9206	8,0497	6,0906	2,2459	2,394	—0,08589
Pt	0,8011	0,3942	4,7643	5,6743	15,677	2,634	2,0491	1,7391
Mg	0,4705	0,0493	2,2446	0,72076	—0,22009	1,4582	0,18465	0,078827
Sn	1,1673	1,1482	3,3443	2,1237	0,047028	2,0234	0,37292	0,24017
Ag	1,4078	1,9230	4,2489	4,7586	3,2775	2,4618	1,0956	0,62151
Zn	1,2326	1,2782	4,0653	4,2969	2,4556	2,3732	0,98194	0,53796
Sb	1,3147	1,6367	2,3329	—1,0353	—0,023972	1,5557	—0,51577	0,179
V	0,6511	0,2051	1,8784	—0,44865	—0,19492	1,2869	—0,21908	0,069473
Nb	0,3176	—0,0144	2,7073	2,205	0,96843	1,6787	0,70413	0,2482
Ta	0,7120	0,2749	2,7057	1,3655	—0,17432	1,686	0,30081	0,11195
Mo	0,6508	0,2034	2,487	0,92009	—0,27769	1,5838	0,19861	0,084946
W	0,6738	0,2331	2,3824	0,62205	—0,33734	1,5369	0,11185	0,0713
Co	—0,1627	—0,1017	3,2762	5,0707	1,8339	1,976	1,6809	—0,1007
Th	0,8111	0,4087	1,4824	—1,4803	0,3644	1,1182	—0,5877	0,1699
U	1,0383	0,8435	2,9404	1,2554	—0,34692	1,8236	0,18638	0,12971
Zr	0,8089	0,4105	0,85405	—2,0773	1,0133	0,76438	—0,88348	0,33723

TABLE 3

Material	$\bar{l}\cdot10^{-11}$, dyne/cm²	$\bar{m}\cdot10^{-11}$, dyn/cm²	$\bar{n}\cdot10^{-11}$, dyn/cm²	$\bar{A}\cdot10^{-11}$, dyn/cm²	$\bar{B}\cdot10^{-11}$, dyn/cm²	$\bar{C}\cdot10^{-11}$, dyn/cm²
Fe	$-3{,}48\pm6{,}5$	$-103{,}0\pm7{,}0$	110 ± 110	110 ± 110	-158 ± 62	$123{,}2\pm68{,}5$
Al	—	—	$-31{,}2\pm1{,}2$	$-31{,}2\pm1{,}2$	—	—

TABLE 4

Material	b, Å	Material	b, Å	Material	b, Å	Material	b, Å
Fe	2,48	Be	2,24	Ag	2,89	Mo	2,72
Al	2,86	Au	2,88	Zn	2,66	W	2,74
Cu	2,56	Cd	2,98	Sb	—	Co	2,51
Ni	2,49	Pt	2,77	V	2,63	Th	3,60
Pb	3,49	Mg	3,21	Nb	2,86	U	—
Ti	2,95	Sn	3,02	Ta	2,86	Zr	3,20

$$\rho^0 E(I_1, I_2, I_3, S) = \left[-2\mu I_2 + \frac{1}{2}\left(K + \frac{4}{3}\mu\right) I_1^2\right] + \\ + \left\{\bar{n} I_3 - 2\bar{m} I_1 I_2 + \frac{1}{3}(2\bar{m} + \bar{l}) I_1^3\right\} + \ldots \tag{3.4}$$

Here

$$I_1 = \mathrm{Sp}\, u_{ik}, \quad I_2 = \mathrm{Sp}\, u_{ik}u_{kl}, \quad I_3 = \mathrm{Sp}\, u_{ik}u_{kl}u_{lm}, \\ u_{ik} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_k} + \frac{\partial u_k}{\partial x_i} + \frac{\partial u_l}{\partial x_i}\frac{\partial u_l}{\partial x_k}\right), \tag{3.5}$$

where u_{ik} are the strain tensor components, and u_i are displacement components. In addition to the parameters $\bar{m}$, $\bar{n}$, $\bar{l}$ defined by the formulas

$$\rho^0\left(\frac{\partial^2 E}{\partial I_1 \partial I_2}\right)_S = -4\bar{m}, \quad \rho^0\left(\frac{\partial^3 E}{\partial I_1^3}\right)_S = 4\bar{m} + 2\bar{l}, \\ \rho^0\left(\frac{\partial E}{\partial I_2}\right)_S = -2\mu, \quad \rho^0\left(\frac{\partial E}{\partial I_3}\right)_S = \bar{n}, \quad \rho^0\left(\frac{\partial^2 E}{\partial I_1^2}\right)_S = K + \frac{4}{3}\mu, \tag{3.6}$$

the elastic moduli $\bar{A}$, $\bar{B}$, $\bar{C}$ related to $\bar{l}$, $\bar{m}$, and $\bar{n}$ by the formulas

$$\bar{l} = \bar{B} + \bar{C}, \ \bar{A} = \bar{n}, \ \bar{m} = \bar{A}/2 + \bar{B}, \\ \bar{B} = \bar{m} - \bar{n}/2, \ \bar{C} = \bar{l} - \bar{m} + \bar{n}/2,$$

are often used. Values of $\bar{A}$, $\bar{B}$, and $\bar{C}$ are presented in Table 3 for a number of metals. Formulas (3.4)-(3.6) of the second-approximation nonlinear theory of elasticity (five-constant theory of elasticity) permit comparing the expressions for the energies $E(I_1, I_2, I_3, S)$ and $E(\varepsilon, D, \Delta, S)$.

To do this, the expressions for the invariants I_1, I_2, I_3 and ε, D, and Δ must be compared in the principal strain axes:

$$\varepsilon = -I_1 + I_1^2 + 2I_2 - \frac{4}{3}I_1^3 + 4I_1 I_2 - 4I_3, \\ D = \frac{1}{3}I_1^2 - I_2 - \frac{7}{3}I_1 I_2 + \frac{2}{3}I_1^3 + 3I_3,$$

$$\Delta = \frac{2}{27} I_1^3 - \frac{1}{3} I_1 I_2 + I_3.$$

Setting

$$\overline{Q} = \frac{\partial E}{\partial D}\Big|_{\varepsilon=\Delta=0}, \qquad \overline{R} = \frac{\partial^2 E}{\partial \varepsilon^2}\Big|_{\varepsilon=D=0},$$
$$\overline{L} = \frac{\partial^2 E}{\partial \varepsilon \partial D}\Big|_{\varepsilon=D=0}, \qquad \overline{M} = \frac{\partial E}{\partial \Delta}\Big|_{\varepsilon=D=0},$$
$$\overline{N} = \frac{\partial^3 E}{\partial \varepsilon^3}\Big|_{\varepsilon=D=0},$$

we obtain the relationship

$$\frac{2\mu}{\rho^0} = \overline{Q}, \quad \frac{\overline{A}}{\rho^0} = 3\overline{Q} + \overline{M}, \quad \frac{K + 4/3\mu}{2\rho^0} = \frac{\overline{Q}}{3} + \frac{\overline{R}}{2},$$
$$-\overline{A} - 2\overline{B} = -\frac{7}{3}\overline{Q} - 2\overline{R} + \frac{1}{2}\overline{L} - \frac{1}{3}\overline{M},$$
$$\frac{\overline{A} + 3\overline{B} + \overline{C}}{3} = \frac{2}{3}\overline{Q} + \overline{R} - \frac{1}{6}\overline{L} - \frac{1}{6}\overline{N} + \frac{2}{27}\overline{M}$$

between the coefficients of the five-constant theory of elasticity and the values of the interpolation coefficients of (2.3). Namely, by expanding the energy E_p in a series in ε, D, Δ

$$E_p = \overline{Q}D + \frac{\overline{R}}{2}\varepsilon^2 + \overline{M}\Delta + \frac{1}{2}\overline{L}\varepsilon D + \frac{1}{6}\overline{N}\varepsilon^3 + \ldots, \tag{3.7}$$

we find

$$\overline{Q} = 2d_0, \quad \overline{R} = a_0^2, \quad \overline{L} = 2d_0^2 l_3, \quad \overline{N} = 3a_0^2 l_1,$$

which means

$$d_0^2 = \mu/\rho^0, \quad \overline{M} = \frac{\overline{A} - 6\mu}{\rho^0}, \quad \frac{K}{\rho^0} = a_0^2,$$
$$l_1 = 2 - \frac{1}{K}\left(\frac{11}{27}\overline{A} + \overline{B} + \frac{\overline{C}}{3} + \frac{\overline{\mu}}{3}(l_3 - 8)\right), \qquad l_3 = \frac{8}{3} + \frac{2}{\mu}\left(K - \frac{\overline{A}}{3} - 2\overline{B}\right). \tag{3.8}$$

The expressions obtained permit utilization of a simplified cold energy formula computed from the second-approximation theory of elasticity

$$h(\varepsilon) = \frac{K}{2\rho^0}\varepsilon^2(1 + l_1\varepsilon), \; d^2(\varepsilon) = \frac{\mu}{\rho^0}(1 + l_3\varepsilon),$$

in (2.1) and (2.2), where the coefficients l_1 and l_3 are found from (3.7) and (3.8).

§4. A change in the internal energy of a material because of a change in its structure must be taken into account in constructing models of an elastoplastic medium when the inelastic strains is accompanied by a change in the internal structure of the medium. In this case, the strain processes will not be reversible, in contrast to the elasticity. In other words, if the absence of external heat inflows $dq^{(e)} = 0$ is assumed, then the relationship

$$\rho T dS = dq' > 0, \tag{4.1}$$

holds for physically infinitesimal particles of the medium, where dq' is the uncompensated heat [2].

We define the uncompensated heat by the following relationship:

$$dq' = \gamma \frac{\sigma_{ij}}{\rho} d\varepsilon_{ij}^p, \tag{4.2}$$

where γ is the fraction of the elementary work in the plastic deformations which goes over into heat, and ε_{ij} is the plastic deformation tensor.

If there is no external heat inflow, then the heat inflow equation is

$$\rho dE = \sigma_{ij} d\varepsilon_{ij}, \tag{4.3}$$

where we take for ε_{ij}

$$d\varepsilon_{ij} = d\varepsilon_{ij}^{e} + d\varepsilon_{ij}^{p}.$$

Let us rewrite (4.3) in the form*

$$\rho\left(\frac{\partial E}{\partial \varepsilon_{ij}^{e}} d\varepsilon_{ij}^{e} + \frac{\partial E}{\partial S} dS + \frac{\partial E}{\partial n_{ij}} dn_{ij}\right) = \sigma_{ij} d\varepsilon_{ij}^{e} + \gamma\sigma_{ij} d\varepsilon_{ij}^{p} + (1-\gamma)\sigma_{ij} d\varepsilon_{ij}^{p}. \tag{4.4}$$

Taking account of (4.1) and (4.2), we obtain the following:

$$T = \frac{\partial E}{\partial S}, \quad (1-\gamma)\sigma_{ij}\frac{d\varepsilon_{ij}^{p}}{dt} = \rho\frac{\partial E}{\partial n_{ij}}\frac{dn_{ij}}{\partial t}.$$

besides (1.3) from (4.4). This last relationship can be used to determine γ

$$\gamma = 1 - \left(\rho\frac{\partial E}{\partial n_{ij}}\frac{\partial n_{ij}}{\partial t}\right)\Big/\left(\sigma_{ij}\frac{d\varepsilon_{ij}^{p}}{dt}\right). \tag{4.5}$$

Therefore, the question of the redistribution of the work in plastic deformation between heat and defect formation is resolved.

The relationship

$$\frac{dW_p}{dt} = \sigma_{ij}\frac{d\varepsilon_{ij}^{p}}{dt},$$

in which the plastic strain rate tensor remains undefined, is used for the rate of change of the work of plastic deformation. The definition of the plastic strain rate tensor depends on the choice of the physical model. A relationship of the type

$$\frac{d\varepsilon_{ij}^{p}}{dt} = J_{ij}(\sigma_{ij}, n_{ij}, T), \tag{4.6}$$

can be assumed in a number of cases, where J_{ij} is the flux density tensor of the structure defects (or the flux density tensor of the dislocations [1]).

To determine the tensor n_{ij}, the kinetic equations must be written by starting from definite physical considerations [6, 7]

$$dn_{ij}/dt = f_{ij}(\sigma_{ij}, n_{ij}, T).$$

Let us examine the problem of the deformation of a long slender rod from low-carbon steel under the constant stress $\sigma_{11} = \text{const}$ (creep). Let us use the Orowan relationship [8, 9] as the dependence (4.6), $d\varepsilon_{11}^{p}/dt = \frac{4}{3} bnu$, where u is the mean dislocation rate.

Assuming the dislocation density of a polycrystalline material to be described satisfactorily by the scalar parameter n, let us use the test dependence [9]

$$\frac{dn}{dt} = \frac{4}{3} m \frac{d\varepsilon_{11}^{p}}{dt}, \tag{4.7}$$

where m is a dislocation multiplication factor.

The values of the coefficient m, borrowed from [10], are presented in Table 5.

The relationship (4.5) for the elastic energy equation presented in Sec. 2 is rewritten in the form

$$\gamma = 1 - \rho d^2 b^2 \frac{dn}{dt}\Big/\sigma_{11}\frac{d\varepsilon_{11}^{p}}{dt}.$$

*The formulas (4.4) are also valid in considering finite deformations if the definition given for the elastic strain tensor in Sec. 2 and the Murnaghan formulas for the stresses are hence used; the relationships (4.6) are true for plastic deformations.

TABLE 5

Material	$m \cdot 10^{-9}$, cm^{-2}	Material	$m \cdot 10^{-9}$, cm^{-2}
Al	80	Mo	80—90
Cu	50	U	400
Ni	8	Fe	33—250

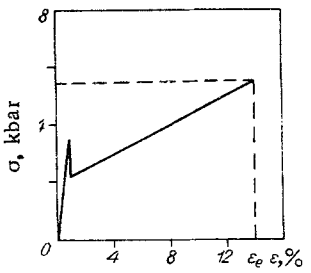

Fig. 1

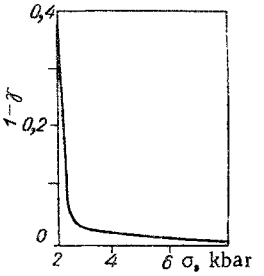

Fig. 2

Taking account of the relationship (4.7), we obtain

$$\gamma = 1 - \rho d^2 b^2 \frac{4}{3} m/\sigma_{11}. \qquad (4.8)$$

Values of $m = 10^{-12}$ cm^{-2} are obtained in [8, 9] for low-carbon steel parameters and $b = 2.5 \cdot 10^{-8}$ cm, $\mu = \rho d^2 = 906$ kbar are used. Substituting these values into (4.8), we obtain $1 - \gamma = 0.302$ for σ_{11} 2.5 kbar, which corresponds to the data of calorimetric measurements.

If the quasistatic strain of a specimen is accompanied by hardening (Fig. 1), and defect formation starts with a certain σ_S, then the value $(1 - \gamma)$ will diminish as σ_{11} increases (Fig. 2), where $1 - \gamma = 0.15$ for $\sigma_{11} = 5$ kbar. It is shown in [9] that the dislocation model used above also holds for shocks of comparatively low intensity $\sigma_{11} \approx 50$ kbar (for high intensity shocks the mechanism of conservative slip of the dislocations will already not predominate). Hence, to compute the quantity $1 - \gamma$ in the case of shock propagation in low-carbon steel rods, (4.8) can be used, from which it follows that the main dissipation because of defect formation occurs on the shock front where the pulse grows from the value σ_S (elastic forerunner) to the final σ_l on the plastic wave front.

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